PV redistribution by localised transient forcing in the shallow-water model

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Motivation: ocean eddy parameterisations

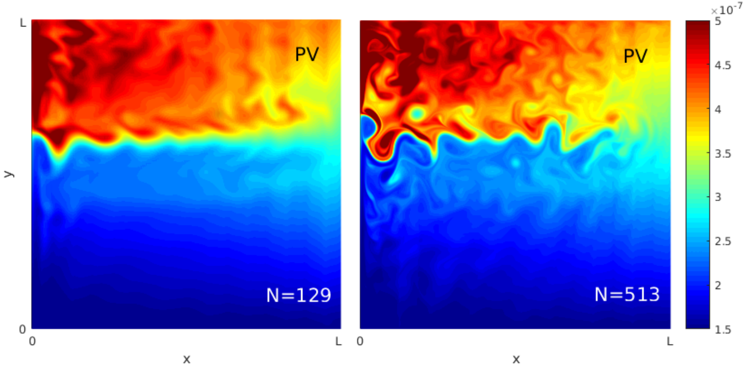

Figure 1: Upper-layer shallow-water potential vorticity in a 3-layer ocean basin. Left: low resolution (poorly resolved eddies). Right: high resolution (somewhat resolved eddies).

The problem

- GCMs commonly employ a diffusive parameterisation for missing eddy effects, which doesn't perform well in jet regions.
- Our approach is to develop an **understanding of the effects of transient, localised forcing** in a simplified shallow-water model, and we look specifically at **how PV is redistributed in the system**.
- The forcing is intended to be an **elementary representation of eddy flux divergences**.
- An understanding of the effects of localised forcing can be applied to form a parameterisation of eddy PV fluxes.

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The 1-L shallow-water model

The non-dimensional equations are

$$Ro\frac{\mathrm{D}u}{\mathrm{D}t} - fv = -\frac{\partial\eta}{\partial x} + \frac{Ro}{Re}\nabla^2 u - \gamma u + Ro\,F_1, \qquad (1)$$

$$Ro\frac{\mathrm{D}v}{\mathrm{D}t} + fu = -\frac{\partial\eta}{\partial y} + \frac{Ro}{Re}\nabla^2 v - \gamma v + Ro\,F_2, \qquad (2)$$

$$\frac{\mathrm{D}\eta}{\mathrm{D}t} + \eta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = F_3. \qquad (3)$$

β-plane Coriolis parameter; Laplacian viscosity; linear friction.

We have two horizontal velocities (u, v) and a sea-surface height η.

F_i $(i = 1, 2, 3)$ are external forcing terms.

Simplifying the system

We seek **periodic** (in time) solutions to the **linearised** system.

Linearisation:

$$u(x,y,t) \to U_0(y) + u'(x,y,t), \quad v(x,y,t) \to 0 + v'(x,y,t),$$
$$\eta(x,y,t) \to H_0(y) + \eta'(x,y,t).$$

It is imposed that the background state is in geostrophic balance.

Periodicity:

$$u'(x,y,t) \to u'(x,y)\exp(2\pi i\omega t)$$

for some forcing frequency ω.

Simplifying the system

The linearised, time-periodic shallow-water system after implementing the Fourier transform in x:

$$\left[i\delta Ro + 4\pi^2 k^2 \frac{Ro}{Re} + \gamma\right]\tilde{u} - \frac{Ro}{Re}\frac{\partial^2 \tilde{u}}{\partial y^2} + \left[Ro\frac{\mathrm{d}U_0}{\mathrm{d}y} - f\right]\tilde{v} + 2\pi i k\tilde{\eta} = \tilde{F}_1,$$

$$\left[i\delta Ro + 4\pi^2 k^2 \frac{Ro}{Re} + \gamma\right]\tilde{v} - \frac{Ro}{Re}\frac{\partial^2 \tilde{v}}{\partial y^2} + f\tilde{u} + \frac{\partial \tilde{\eta}}{\partial y} = \tilde{F}_2,$$

$$i\delta\tilde{\eta} + 2\pi i k H_0\tilde{u} + \frac{\mathrm{d}H_0}{\mathrm{d}y}\tilde{v} + H_0\frac{\partial \tilde{v}}{\partial y} = \tilde{F}_3,$$

where $\delta(k; y; \omega) = 2\pi(\omega + U_0 k)$ and $\tilde{\cdot}$ denotes a Fourier transform in x with zonal wavenumbers k.

The forcing

Intended to be an **elementary representation of mesoscale eddy flux divergences**, the F_3 forcing term is

$$F_3(x,y) = \begin{cases} \frac{Af}{2}\left(1+\cos\left(\pi\frac{r}{r_0}\right)\right) - \varepsilon & \text{for} \quad r \le r_0, \\ -\varepsilon & \text{for} \quad r > r_0, \end{cases}$$

where $r = \sqrt{x^2 + (y - y_0)^2}$, r_0 is the radial extent of the forcing.

Momentum forcing terms are defined via geostrophic balance:

$$F_1 = -\frac{1}{f}\frac{\partial F_3}{\partial y} \quad \text{and} \quad F_2 = \frac{1}{f}\frac{\partial F_3}{\partial x}.$$

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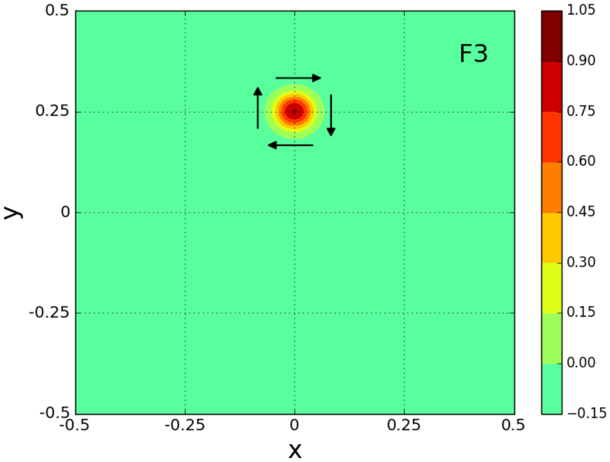

Figure 2: Forcing on the continuity equation. Arrows indicate direction of momentum forcing.

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Summary of the system

- 1L shallow-water system linearised about zonal flow, set-up in a channel with northern/southern boundaries.
- We seek periodic solutions (example period 60 days) to periodic forcing (an assumption which can be relaxed to general forcing time-dependence).
- The forcing is intended to represent transient eddy flux divergences.

Some parameters: basin size, $L = 3840$ km; system periodicity, $T = 60$ days; forcing radius, $r_0 = 90$ km, Rossby number, $Ro = 3.1 \times 10^{-5}$; Reynolds number, $Re = Re_0 = 384$; grid resolution, $dx = 7.5$ km.

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Typical solutions: uniform zonal background flow

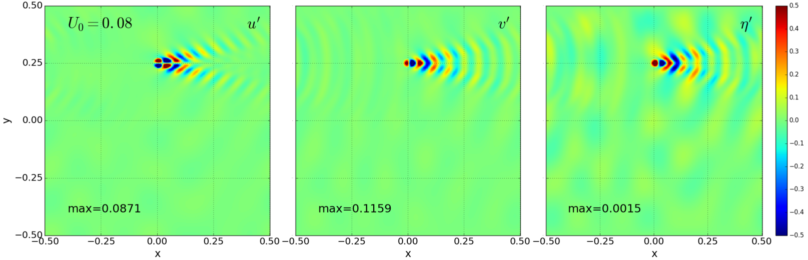


Figure 3: Snapshot of typical shallow-water solutions with a uniform background flow $U_0 = 0.08$ m s^{-1}. Zonal velocity anomaly (left), meridional velocity anomaly (center) and sea-surface height anomaly (right).

Defining potential vorticity

The shallow-water potential vorticity (PV) is defined as

$$q = \frac{\zeta + f}{\eta}$$

where $\zeta = \partial_x v - \partial_y u$ is the relative vorticity.

The footprint, defined as the **time-mean PV flux convergence** is:

$$P = -\nabla \cdot \left(\overline{\mathbf{u}q}\right).$$

This may be interpreted as a cumulative PV forcing if the system were nonlinear (Berloff, 2015).

PV flux convergence

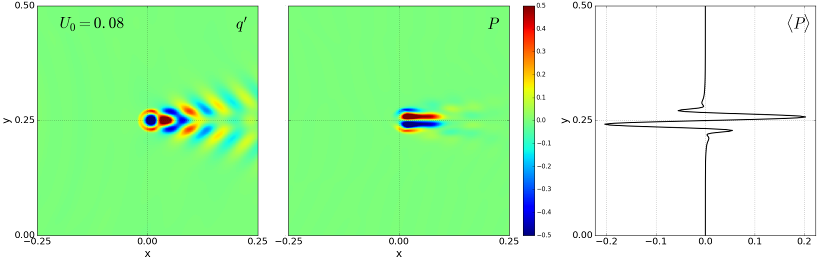


Figure 4: Snapshot of PV (left), time-mean PV flux convergence (center), and time-mean zonal-mean PV flux convergence (right) for a uniform background flow $U_0 = 0.08$ m s^{-1}.

PV flux convergence

- Here, we approximate the nonlinear PV flux convergence using linear solutions (quasi-linear approximation). Waterman & Jayne (2012) and Haidvogel & Rhines (1983) produce fully nonlinear P in the QG system and find very similar spatial patterns as P presented here.
- For uniform BG flows (we've tested the range $[-0.3, 0.5]$ m s^{-1}), we **consistently observe dipole footprint structure** with northward PV flux convergence.
- The footprints spatial structure can be explained by considering the shape of the 'eddies' in our solutions.

The role of eddy shape

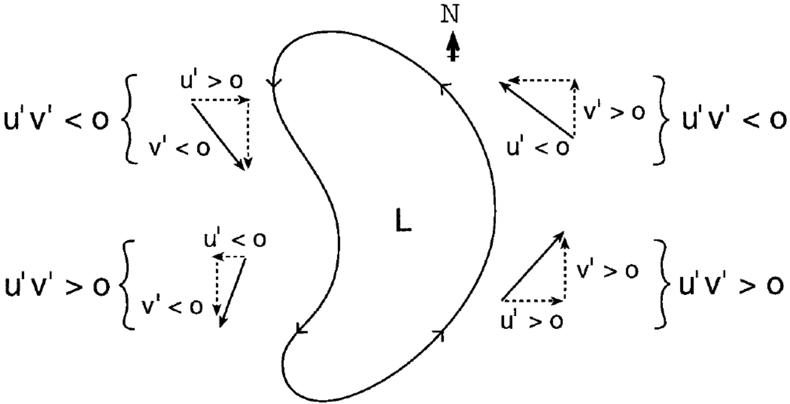
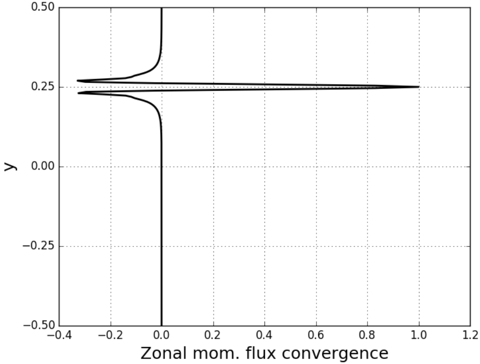


Figure 5: Schematic of momentum fluxes in a bow-shaped eddy (Wardle and Marshall, 2000).

The role of eddy shape

The bow-shaped flow disturbances propagating from the forcing location **drive eastward momentum convergence** at the forced latitudes, corresponding to **northwards PV flux convergence**.



Equivalent eddy fluxes (EEFs)

We want to understand the properties of P, specifically, how does it depend on uniform U_0?

We define the EEF in two components:

$$E_N \equiv L_N \int_{y>y_0} \langle P \rangle \, dy$$

where

$$L_N = \frac{\int_{y>y_0} |y-y_0| \, |\langle P \rangle| \, dy}{\int_{y>y_0} |\langle P \rangle| \, dy}.$$

The equivalent southern quantity E_S, is defined with $y < y_0$ in the integration limits. Then, define the EEF as follows:

$$E \equiv E_N - E_S.$$

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EEF vs U0

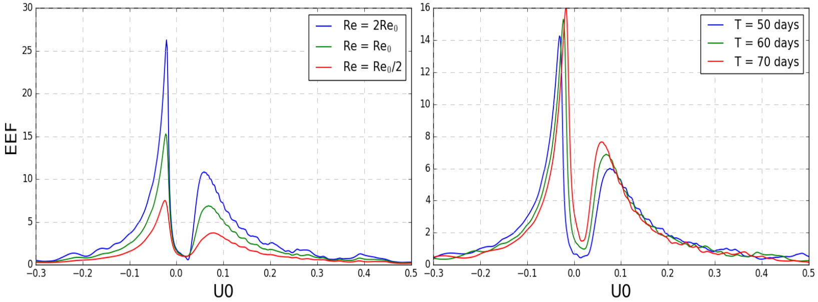

Figure 6: Plots of the EEF versus uniform background flows U_0 (units m s^{-1}). Left: Reynolds number Re is varied from reference value $Re = 384$. Right: forcing solution period is varied (units days).

EEF properties

The EEF is robustly maximised for weak westwards flow $U_0 = U_w \approx -0.02$ m s^{-1}, and for eastward flows $U_0 = U_e \approx 0.07$ m s^{-1}. Minimised for weak eastward flow $U_0 = U_m = 0.02$ m s^{-1}. Why is this?

The primary reason for this is resonance with channel modes.

Eddy shape, which is dependent on U_0, may also play a role in the behaviour of the EEF.

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Resonance with channel modes: decomposition

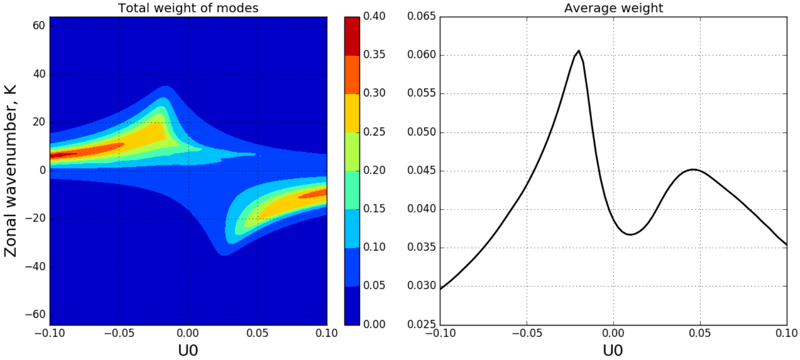

Figure 7: Eigenmode decompositions. Left: eigenmode weight distribution among zonal wavenumbers. Right: average weight in solutions versus uniform background flow U_0.

Eddy-eddy interaction

The term $u'_{yy}v'$ provides the dominant contribution the zonal-mean footprint.

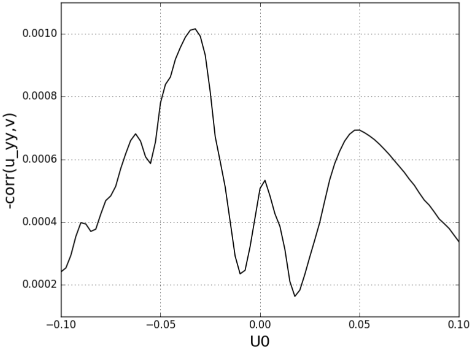

Figure 8: Negation of the time-mean correlation between v' and u'_{yy} in a 0.5 x 0.5 box centered on the forcing location.

Gaussian jet background flow

We can use inhomogeneous background flows. Define a Gaussian-jet BG flow as

$$U_0(y) = a\left(\exp\left(-\frac{y^2 - \frac{1}{2}}{2\sigma^2}\right) - 1\right),$$

where a is the amplitude of the jet and $\sigma > 0$ is the jet width.

Max jet speed, $U_{\text{max}} = 0.8$ m s^{-1}, jet width, $\sigma \approx 75$ km.

EEFs for Gaussian-jet BG flow: reference jet

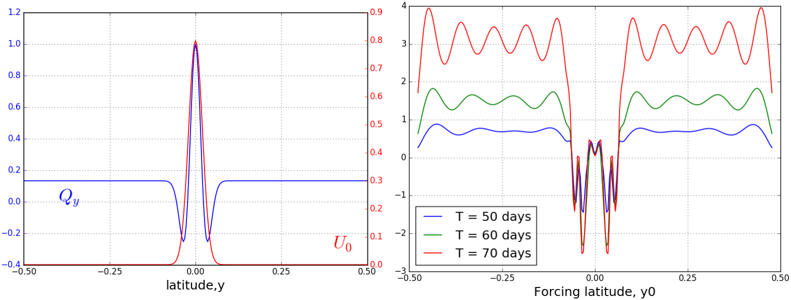

Figure 9: Left: zonal Gaussian jet background flow U_0 (red) and background meridional PV gradient (blue). Right: EEF against forcing latitude y_0.

EEFs for Gaussian BG flow: wide jet

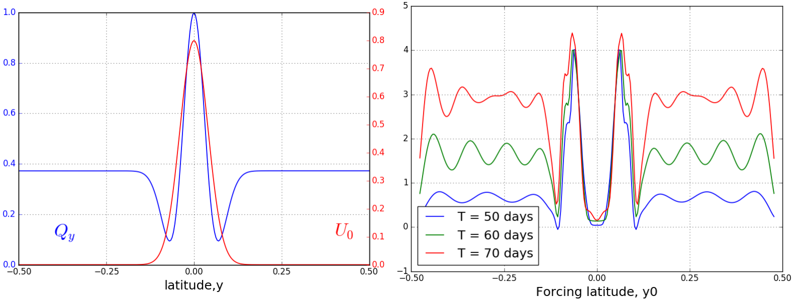

Figure 10: Left: zonal Gaussian jet background flow U_0 (red) and background meridional PV gradient (blue). Right: EEF against forcing latitude y_0.

Application towards parameterisations

- In Berloff (2015), a parameterisation of eddy PV fluxes was implemented using this approach.
- Depending on the local background flow, external dipole inputs of PV were included a coarse resolution model.
- Resulting simulations exhibit coherent eastward jet extension otherwise absent in coarse-resolution model.

Application towards parameterisations

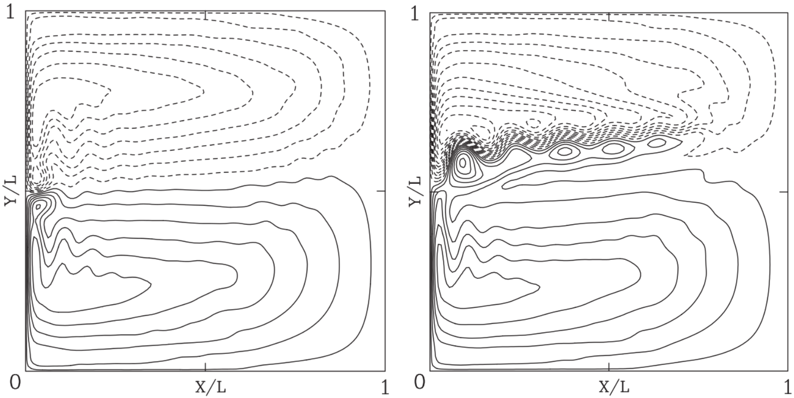

Figure 11: Left: streamfunction contours in low resolution wind-driven QG simulation (Berloff 2015). Right: the same simulation with external inputs of PV and subsequent rectified jet.

Future steps

- Look deeper into the role of eddy shape with regards to the EEF.
- Extend the analysis into the 2-layer system, which requires defining a suitable forcing regime.
- Consider a wider range of Gaussian-jet background flows.

Berloff, P., 2015. Dynamically consistent parameterization of mesoscale eddies. Part 1: Simple model. Ocean Modelling 87, 1-19.

Haidvogel, D. & Rhines, P. 1983. Waves and circulation driven by oscillatory winds in an idealized ocean basin. Geophys. Astrophys. Fluid Dyn. 25, 163.

Wardle, R. & Marshall, J. 2000. Representation of eddies in primitive equation models by a pv flux. J. Phys. Oceanogr. 30, 24812503.

Waterman, S. & Jayne, S. 2012. Eddy-driven recirculations from a localized transient forcing. J. Phys. Oceanogr. 42, 430447.